Some Aspects of the Elastic Anisotropy of Solids

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Abstract

In this thesis, new aspects of the anisotropy of the Poisson's ratio, shear modulus and biaxial modulus have been investigated. This study also includes the evaluation of an existing method of determining the Young's modulus of anisotropic materials using indentation measurements. Further, a new method to determine the elastic constants of cubic crystals from indentation measurements alone is proposed.

When a property or phenomenon varies with the direction of measurement, it is said to be anisotropic. Elastic properties of single crystals, layered/fibre-reinforced composites and textured polycrystals are few common examples of anisotropic materials. Random polycrystals, glasses and other amorphous materials are generally isotropic. The anisotropy of a material properties can be characterised using their tensor descriptions. The elastic response of linear elastic solids is governed by the Hooke's law. Stresses and strains are second-rank tensors while the stiffness/compliance constants are fourth-rank tensors.

Neumann's principle states that the symmetry elements of any physical property of a crystal must include the symmetry elements of the point group of the crystal. Therefore, the anisotropy of elastic properties of crystals are closely related to its symmetry. The anisotropy of properties like the Young's modulus (E), shear modulus (G) and Poisson's ratio (ν) have been extensively investigated. When the loading direction is confined within a plane with a three- or six-fold rotational symmetry, the Young's modulus has been reported to be invariant throughout the plane. The Poisson's ratio and shear moduli are also known to be invariant when the transverse plane and shear plane, respectively, possess a three-, four-, or six-fold symmetry. In the current study, it is found that transverse/shear planes without symmetry can also have invariant Poisson's ratio/shear modulus. Conditions on the elastic constants for the occurrence

of such planes are identified and expressions for such plane normals are derived for all crystal systems (*Shrikanth et al., 2020, International Journal of Solids and Structures, Shrikanth et al., under review, Mechanics of Materials*).

The anisotropy in properties is observed in all forms of crystalline materials. Crystalline thin films deposited on substrates develop stresses and strains within the film. These stresses can be high and may result in the formation of defects or cracks. The actual states of stresses and strains within a film are not easily measurable. Hence, the biaxial modulus (B) is defined as the ratio of the in-plane tensile stress to the corresponding longitudinal strain under a state of equibiaxial strain or equibiaxial stress. It also appears in the Stoney equation which is used to determine the stresses in thin films from the measurement of its curvature. In the case of isotropic films, the biaxial modulus is given by $E/(1-v)$, and the states of equibiaxial strain and stress are equivalent. For anisotropic films, it is important to use the appropriate expressions for the biaxial modulus while determining the film stresses. The expressions for standard film orientations such as {001}, {110} and {111} of cubic films are known. In the current study, expressions for the biaxial moduli of thin films with a general (*hkl*) orientation are derived. An in-depth study on the case of cubic thin films is also carried out (*Shrikanth et al., 2022, Thin Solid Films*). The global extrema are identified and the expressions for the shear strains and out-of-plane normal strain are derived. The significance of the choice of boundary conditions is demonstrated by comparing the biaxial modulus derived under a zero shear strain state with that under a zero shear stress state.

In many cases, the deposited thin films possess a fiber texture. The polycrystalline averages of their elastic properties provide useful information. The Voigt and Reuss averages are known to be the upper and lower bounds of the polycrystalline moduli. The case of coinciding Voigt and Reuss bounds imply that the polycrystal deforms with identical states of stresses and strains, similar to the case of single crystals. In the present study, it is shown that the bounds of the biaxial modulus can coincide even when the fiber axis does not possess rotational symmetry. Film planes within which the biaxial modulus remains independent of the direction of measurement are also identified for all crystal systems. The results may be found in *Shrikanth et al.* (*2021, Journal of Applied Physics*).

The anisotropy of small-scale deformation processes can be studied using nanoindentation. Nanoindentation tests give a load-displacement (*P-h*) curve from which the contact stiffness can be estimated. The indentation modulus (*M*) is a property of the indented material and the geometry of the indenter. The indentation modulus of isotropic materials is the same as its plane strain Young's modulus which is given by $E/(1-\nu^2)$. In the case of anisotropic materials, *M* is a complicated function of the indented plane, elastic constants of the indented material, and the indenter geometry. A common practice is to use the isotropic equation with the measured value of *M* and an assumed value of the bulk Poisson's ratio to determine the Young's modulus of anisotropic materials along the direction of indentation. The validity of such an approximation is examined in this study (*Shrikanth et al., 2022, Journal of Materials Research*). It is found that $E_{(hkl)} = M_{(hkl)}(1-\nu^2_{isotropic})$ is highly likely to give reasonable results when the indented plane is either cubic {110} or any prismatic plane of hexagonal materials. In the case of other planes, the need to use appropriate equations accounting for the anisotropy is emphasised.

The elastic (stiffness/compliance) constants provide complete information regarding the elastic deformation response of solids. Relatively convenient and reliable methods like resonant ultrasound spectroscopy and Brillouin scattering are applicable to single crystalline specimens alone. Most of the materials with useful applications are polycrystals. However, simple, and effective methods to extract the elastic constants from polycrystalline specimens are not well established. Methods that combine nanoindentation measurements with electron backscatter diffraction (EBSD) have been developed to extract the elastic constants. Such methods either involve complex computations, use approximate relationships, or require the value of at least one bulk elastic property to be known/assumed. In the current study, approximate closed form expressions, with improved accuracy, are derived for the indentation moduli on the {100}, {110} and {111} planes of cubic materials. On identifying grains with these orientations within the polycrystal using EBSD, the indentation moduli can be measured, and the three independent elastic constants of the cubic crystallites are obtained (*Shrikanth et al., 2022, Journal of Materials Research*).

Publications

1. **Shrikanth, S.**, Neelakantan, S., Prasad, R., 2023. **On the Determination of the Elastic Properties of Anisotropic Materials from Indentation Measurements.** *Journal of Materials Research*, 38(2), 350–367
   https://doi.org/10.1557/s43578-022-00815-8
2. **Shrikanth, S.**, Prasad, R., Knowles, K.M., 2022. **The Biaxial Modulus of Single Crystal Cubic Thin Films under an Equibiaxial Strain**. *Thin Solid Films* 751, 139176
   https://doi.org/10.1016/j.tsf.2022.139176
3. **Shrikanth, S.**, Prasad, R., Neelakantan, S., 2021. **Biaxial modulus in fiber-textured thin films: Coinciding Voigt and Reuss bounds and planes of isotropy**. *Journal of Applied Physics* 129, 215101
   https://doi.org/10.1063/5.0041016
4. **Shrikanth, S.**, Knowles, K.M., Neelakantan, S., Prasad, R., 2020. **Planes of isotropic Poisson's ratio in anisotropic crystalline solids**. *International Journal of Solids and Structures* 191–192, 628–645.
   https://doi.org/10.1016/j.ijsolstr.2019.10.014